Tensors 2020

JEE Main

December 22, 2019

Answer Key

Single Option Correct

Physics	Chemistry	Mathematics
1.(A)	1.(B)	1.(Cancelled)
2.(B)	2.(D)	2.(D)
3.(C)	3.(B)	3.(D)
4.(A)	4.(B)	4.(D)
5.(C)	5.(C)	5.(B)
6.(B)	6.(B)	6.(Cancelled)
7.(C)	7.(C)	7.(A)
8.(C)	8.(A)	8.(D)
9.(D)	9.(All Correct)	9.(A)
10.(A)	10.(D)	10.(B)
11.(A)	11.(B)	11.(A)
12.(D)	12.(D)	12.(C)
13.(B)	13.(A)	13.(D)
14.(B)	14.(C)	14.(A)
15.(Cancelled)	15.(B)	15.(C)
16.(A)	16.(B)	16.(C)
17.(B)	17.(B)	17.(A)
18.(B)	18.(A)	18.(D)
19.(A)	19.(C)	19.(A)
20.(B)	20.(A)	20.(A)

Numerical Answer

Physics	Chemistry	Mathematics
1.(5.00)	1.(2.00)	1.(0.86)
2.(63-64)	2.(6.00)	2.(11.25)
3.(2.00)	3.(6.54)	3.(5.00)
4.(25-26)	4.(6.00)	4.(2.00)
5.(3.00)	5.(4.00)	5.(4.00)

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Solutions

Physics

Single Option Correct

1. (A)
Buoyant force is defined as the upward vertical force applied by a fluid on a body due to pressure difference. Here, since the density of the cone is greater than water, the cone will sink and touch the base of the container. Therefore there will be no water below the cone, hence no upward force by liquid. The cone experiences only downward force due to liquid above it. Thus buoyant force is 0.

2. (B)
consider a laminar body (eg: rectangular plate as in the figure) free to rotate about a horizontal axis as shown in the figure.
From the figure,

$$\text{torque } \tau = \mathrm{F} \cdot \mathrm{d}$$
$$= -mgL\sin\theta$$

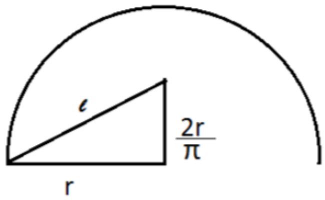
for small values of θ, $\sin\theta = \theta$. So the above equation becomes,

$$\tau = -mgL\theta$$
$$\text{angular acceleration } \alpha = \frac{\tau}{I}$$
$$= \frac{-mgL}{I}\theta$$

which is similar to

$$\alpha = -\omega^2\theta$$
$$\therefore \omega^2 = \frac{mgL}{I}$$
$$\mathrm{T}^2 = \frac{4\pi^2 I}{mgL}$$

Here, we are given a semicircular ring oscillating about a horizontal axis passing through one of its end.
So, moment of inertia $I = mr^2 + mr^2 = 2mr^2$
L can be found from the figure using Pythagoras theorem as the center of mass of a semicircular ring is $\frac{2r}{\pi}$ units from it centre



$$L = \sqrt{r^2 + \left(\frac{2r}{\pi}\right)^2}$$
$$= \frac{r}{\pi}\sqrt{\pi^2+4}$$

Substituting in the equation for T,

$$\mathrm{T} = \frac{8\pi^3 r}{g\sqrt{\pi^2+4}}$$

3. (C)
At maximum current

$$\frac{di}{dt} = 0$$
$$L\frac{di}{dt} = 0$$

hence potential difference across both the capacitors combined is zero

$$\frac{q_3}{C} = -\frac{q_4}{2C}$$

By charge conservation

$$-2q + q = -q_3 + q_4$$
$$q_3 = \frac{q_4}{2}$$

By energy conservation

$$\frac{(2q)^2}{2C} + \frac{q^2}{2(2C)} = \frac{Li^2}{2} + \frac{q_3^2}{2C} + \frac{q_4^2}{4C}$$

Solving these equations, we get,

$$i = \frac{5q}{\sqrt{6LC}}$$

4. (A)
When the key is open,

$$\text{current } I = \frac{\varepsilon_2}{2R + R_A}$$

After key K is closed, let the ammeter current be I_1
Then after appliying KVL along the loop containing both cells,

$$\varepsilon_2 - I_1 R - \varepsilon_1 - I_1 R_A = 0$$

Substituting for ε_2

$$2IR + IR_A - I_1 R - \varepsilon_1 - I_1 R_A = 0$$

Rearranging ,

$$\begin{aligned} \varepsilon_1 - IR &= IR + IR_A - I_1 R - I_1 R_A \\ &= (I - I_1)(R + R_A) \end{aligned}$$

$$\text{i.e., if, } \varepsilon_1 > IR_1$$
$$I_1 < I$$

5. (C)
L=5mH So, $X_L = \omega L = 10\Omega$
C=50μF So, $X_C = \dfrac{1}{\omega C} = 10\Omega$
Since $X_L = X_C$, The circuit is in LC resonance,

$$\begin{aligned} \therefore \text{ Ammeter reading I} &= \frac{V_{rms}}{R} \\ &= \frac{20}{10\sqrt{2}} \\ &= 1.4\text{A} \\ \text{Voltmeter reading V} &= V_L - V_C + V_R \\ &= 0 + 4\text{I} \\ &= 5.6\text{V} \end{aligned}$$

6. (B)
The needle aligns itself in the direction of net magnetic field. Let B be the magnetic field due to coil at a distance x, which is along the axis of coil and hence perpendicular to H. Initially, needle is along H.

$$\therefore, \tan\varphi = \frac{B}{H}$$

We know,

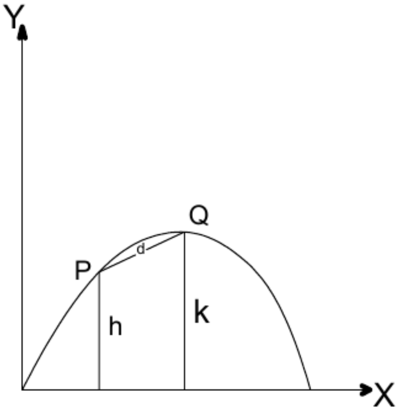
$$B = \frac{\mu_0 n i r^2}{2\left(r^2 + x^2\right)^{\frac{3}{2}}}$$

Therefore,

$$\tan\varphi = \frac{\mu_0 n i r^2}{2H\left(r^2 + x^2\right)^{\frac{3}{2}}}$$

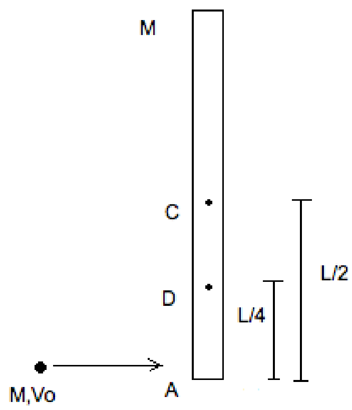
From the equation, we can see that only graph (B) is correct

7. (C)



Making $h = 0,\ d = k$, we get a ball thrown straight up.
Now substitute and check the options for $v = \sqrt{2gk}$ in this case

8. (C)



By COAM about point D,

$$\frac{MV_0L}{4} = \left[\frac{ML^2}{12} + \frac{ML^2}{16} + \frac{ML^2}{16}\right]\omega$$

$$\omega = \frac{6V_0}{5L}$$

$$\begin{aligned} Now, t &= \frac{\theta}{\omega} \\ &= \frac{\pi}{2\omega} \\ &= \frac{5\pi L}{12V_0} \end{aligned}$$

9. (D)

10. (A)

$$
\begin{aligned}
v_x &= v_0 \cos 45^\circ \\
&= \frac{v_0}{\sqrt{2}} \\
v_y &= v_0 \sin 45^\circ - gt \\
&= \frac{v_0}{\sqrt{2}} - v_0 \\
&= v_0 \left[\frac{1-\sqrt{2}}{\sqrt{2}}\right] \\
x &= v_x t \\
&= \frac{v_0^2}{\sqrt{2}g} \\
y &= (v_0 \sin 45^\circ)\, t - \frac{1}{2}gt^2 \\
&= \frac{v_0^2}{2g\left[\sqrt{2}-1\right]} \\
\mathrm{L} &= m\left(\vec{r} \times \vec{v}\right)
\end{aligned}
$$

Substituting the values,

$$\mathrm{L} = -\frac{mv_0^3}{2\sqrt{2}g}\hat{k}$$

11. (A)

$$
\begin{aligned}
\text{Intensity } I &= \frac{1}{2}\varepsilon E^2 c \\
P &= 2\frac{I}{c} \\
&= \varepsilon E^2 \\
E &= \sqrt{\frac{P}{\varepsilon}}
\end{aligned}
$$

We get direction of E from $\vec{c} = \vec{E} \times \vec{B}$

12. (D)
Let us consider two forces acting at symmetrical points about the centre. Since pressure acts normally to the surface, the forces will be perpendicular to the curved surface and will pass through the centre. We can resolve the forces into vertical and horizontal components. The net horizontal force will be along OX and the net vertical force will be along OY since pressure increases as depth increases. Therefore the net force is neither along OX nor along OY. Thus the net force may or may not be along OA.

13. (B)

$$\frac{d\theta}{dt} \propto \frac{A}{m}$$

$$\text{Time taken} \propto \frac{m}{A}$$

As volume doubles and density remains the same, mass also doubles.

$$
\begin{aligned}
\text{So, } V' &= 2V \\
\frac{4}{3}\pi R'^3 &= \frac{4}{3}\pi R^3 \times 2
\end{aligned}
$$

$$
\begin{aligned}
\text{So, } R' &= R \times 2^{\frac{1}{3}} \\
Area &= 4\pi R^2 \times 2^{\frac{2}{3}}
\end{aligned}
$$

$$
\begin{aligned}
\frac{T}{T'} &= \frac{m}{m'} \times \frac{A'}{A} \\
&= \frac{1}{2} \times 2^{\frac{2}{3}} \\
&= 2^{-\frac{1}{3}} \\
T' &= T \times 2^{\frac{1}{3}} \\
&= 27 \times 2^{\frac{1}{3}}
\end{aligned}
$$

14. (B)

$$C_p - C_v = \frac{R}{M}$$

Where M is the molecular mass

$$C_p - C_v = 0.297$$

$$
\begin{aligned}
\text{So, } M &= 28\text{g} \\
\text{No. of moles } n &= \frac{7}{28} \\
&= 0.25 \\
\text{Internal Energy } &= \frac{nfR\Delta T}{2}
\end{aligned}
$$

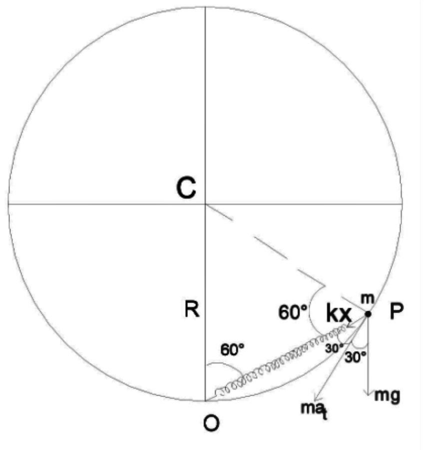
$f = 5$ as it is a diatomic gas
Along path BC,

$$
\begin{aligned}
\Delta U &= 0.25 \times 2.5 \times (-200) \times R \\
&= -125R
\end{aligned}
$$

15. (Question Cancelled)

16. (A)
Theory

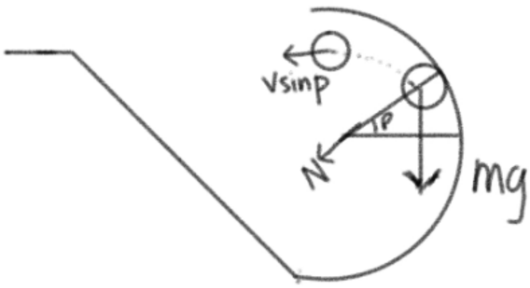
17. (B)



Final length of spring $= R$
Change in length,

$$
\begin{aligned}
x &= R - \frac{3}{4}R \\
&= \frac{1}{4}R \\
kx &= \frac{mgR}{4} \\
F_t &= ma_t \\
&= \frac{mg}{4}\cos 30^\circ + mg\cos 30^\circ \\
&= \frac{5\sqrt{3}}{8}mg \\
a_t &= \frac{5\sqrt{3}}{8}g
\end{aligned}
$$

18. (B)



Centripetal Force is given by,

$$\frac{mv^2}{r} = N + mg\sin p$$

At a point the particle loses its contact, normal reaction is 0

$$
\begin{aligned}
N &= 0 \\
\frac{mv^2}{r} &= mg\sin p \\
r &= \frac{h}{2} \\
v^2 &= \frac{gh\sin p}{2}
\end{aligned}
$$

Using energy conservation

$$mgh = \frac{mgh}{2} \times (1 + \sin p) + \frac{1}{2}mv^2$$

Solving

$$
\begin{aligned}
v &= \sqrt{\frac{gh}{3}} \\
\sin p &= \frac{2}{3} \\
\text{Maximum velocity } &= v\sin p \\
&= \frac{2}{3}\sqrt{\frac{gh}{3}}
\end{aligned}
$$

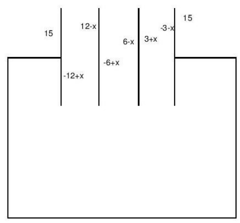
19. (A)
Since only a single beta particle is forming, the mass of atom initially should be greater than mass of Y and electron combined

20. (B)

$$
\begin{aligned}
\text{Angle of the moment of the electrons } &= \frac{nh}{2\pi} \\
mvr &= \frac{nh}{2\pi} \\
\text{de-Broglie wavelength } \lambda &= \frac{h}{mv} \\
&= \frac{2\pi r}{n} \\
&= 2\pi r \qquad \text{(n=1)}
\end{aligned}
$$

Numerical Answer

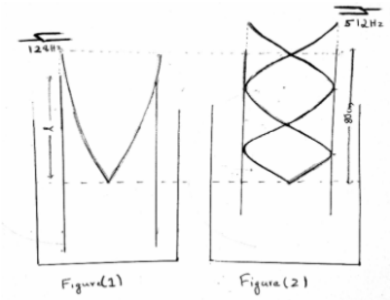
1. (5.00)



If x charge passes through the wire,
Applying kirchhoff's law,

$$
\begin{aligned}
-\frac{12-x}{C} + \frac{6-x}{C} + \frac{-3-x}{C} &= 0 \\
15 - 3x &= 0 \\
x &= 5
\end{aligned}
$$

2. (63-64)



Above figures shows the waves formed in each case
Consider,
f=128 Hz
f'=512 Hz
we have,

$$\text{Velocity} v = f\lambda$$
$$\lambda' = \frac{f}{f'}\lambda$$
$$= \frac{\lambda}{4}$$

So the wavelength when using 512 Hz is one fourth the wavelength when using 128 Hz
From the figures we can see a node is formed at length $\lambda/4$ while using 512 Hz in place we got an Antinode when we used 128 Hz
Now on increasing the length the next resonance is found to be at $\frac{\lambda}{4} + \frac{\lambda'}{4} = \frac{5\lambda'}{4}$ since,

$$\lambda' = \frac{\lambda}{4}$$
$$\frac{5\lambda'}{4} = \frac{5\lambda}{16}$$

from figure(1)

$$l + \text{End Correction} = \frac{\lambda}{4} \qquad (1)$$

from figure (2)

$$80 + \text{End Correction} = \frac{5\lambda}{16} \qquad (2)$$

$$\lambda = \frac{v}{f} = \frac{336}{128}$$

subtracting (1) from (2)

$$80 - l = \frac{\lambda}{16}$$
$$l = 80 - \frac{\lambda}{16}$$
$$= 63.52cm$$

3. (2.00)
Differentiate the equation

$$y = 3x^2 + 7x - 5e^x$$
$$\frac{dy}{dx} = 6x + 7 - 5e^x$$

$\frac{dy}{dx}$ gives the slope of the trajectory at any point.
$\therefore \tan\theta = \frac{dy}{dx}$
At x=0,

$$\tan\theta = 3 \times 0 + 7 - 5e^0$$
$$= 2$$

4. (25-26)
Time taken $\propto$ h^2 (h=depth)
Differential form of time taken

$$\int T\ dt = k\int_{h_1}^{h_2} H\ dh$$

So,from surface T $\propto$ h^2
Whereas from h_1 to h_2, T $\propto h_2^2 - h_1^2$

$$\frac{T}{T'} = \frac{15^2}{x^2 - 15^2}$$
$$= \frac{1}{\left(\frac{x}{15}\right)^2 - 1}$$
$$\frac{T'}{T} = \left(\frac{x}{15}\right)^2 - 1$$
$$\frac{1}{2} + 1 = \left(\frac{x}{15}\right)^2$$
$$x = 18.21$$

5. (3.00)
Half life of B is 1.5 days. There for in three days there would be $\frac{1}{4}$ of initial number of atoms of B. Number of atoms of A remaining is twice of that of B. There for number of atoms of A becomes half in three days.

Chemistry

Single Option Correct

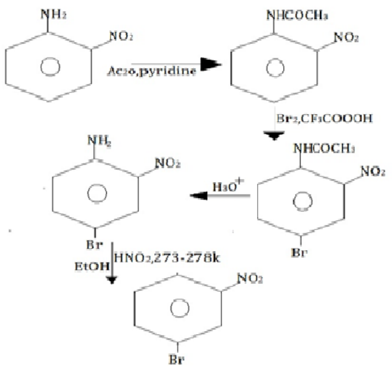
1. (B)
   Correct matches are,

   chloroxylenol — $FeCl_3$ test (test for phenol)
   northindrone→Novestrol — acetate (intermediate)
   sulphobenzamide — Cystiene (both contain sulphur)
   chloramphenicol — azodyetest

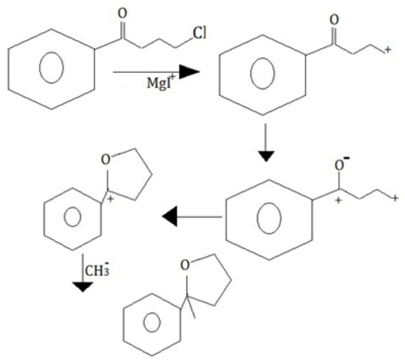
2. (D)
   Fact

3. (B)
   Fact

4. (B)



5. (C)



6. (B)
   Fact

7. (C)
   Fact

8. (A)

   A. Fact

   B. Planar compound

   C. No hybridisation due to Drago's rule

   D. Reverse order

9. (All Choices Correct)

10. (D)
    The given compound is $[CoCl_2(en)_2]Br$, therefore, only AgBr is formed.
    Moles of compound = 0.3
    Moles of $AgNO_3$=M × V=0.1
    Only 0.1 moles of AgBr is formed

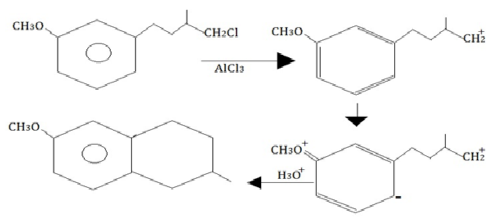
11. (B)

$$K = \frac{0.693}{t_{1/2}}$$
$$K_{653} = 1.925 \times 10^{-3}\text{min}^{-1}$$

$$\log K_{723} - \log K_{653} = \frac{E_a(T_2 - T_1)}{T_1T_2 \times 2.303R} \approx 1.55$$
$$\log K_{723} = -1.1656$$
$$K_{723} = 6.23 \times 10^{-2}\text{min}^{-1}$$

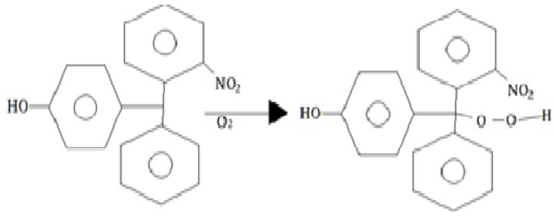
$$t = \frac{2.303}{K}\log\left(\frac{100}{100-75}\right) = 20.33\text{min}$$

12. (D)



13. (A)
    $P_4 + 8SOCl_2 \rightarrow 4PCl_3(A) + 4SO_2(B) + 2S_2Cl_2(C)$

14. (C)



Here OH will be attached to group with most +R so as stabilize oxonium ion in intermediate

15. (B)
$\Delta_f H^\circ$ values of fluoride become less negative as we go down the group, whilst the reverse is true for chlorides, bromides and iodides. For a given metal $\Delta_f H^\circ$ always become less negative from fluoride to iodide

16. (B)
Fact

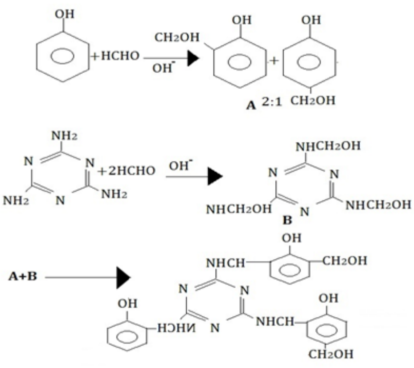
17. (B)

$$\text{Molar Mass of solvent} = 1000\left(\frac{\Delta H_{vap}K_b}{RT_b^2}\right)$$
$$= 64\text{g}$$

$$\text{Mole fraction of solvent} = 0.4$$
$$P_{solvent} = (\chi)P^\circ_{solvent}$$
$$= 140\text{Torr}$$
$$P_{water} = 90\text{Torr}$$
$$P_{total} = 230\text{Torr}$$

$$\chi_{solvent} \text{ in first condensate} = \text{Y}_{solvent}$$
$$= \frac{140}{230}$$
$$= 0.6$$

18. (A)



19. (C)

$$\text{N}_2\text{O}_4 \rightleftharpoons 2\text{NO}_2$$

Given 50% decompositon,

$$P_{N_2O_4} = \frac{1-0.5}{1+0.5}$$
$$= \frac{1}{3}$$

Similarly,

$$P_{NO_2} = \frac{2\times 0.5}{1.5}$$
$$= \frac{2}{3}$$
$$K_p = \frac{(P_{NO_2})^2}{P_{N_2O_4}}$$
$$= \frac{4}{3}atm$$

$$\Delta G = -RT\ln(K_p)$$
$$= -766.90\text{KJ/mol}$$

20. (A)
Fact

Numerical Answer

1. (2.00)
Molar mass of NH_4HS=51g
No. of moles=0.1
$P_{NH_3}=P_{H_2S}=P=\sqrt{K_p}$=0.25atm
So, P_{total}=0.5atm
PV=nRT
So, n_{total}=0.05 and also, n_{total}=(0.25)× degree of dissociation
$\implies$ Degree of dissociation = 0.25
$\implies$ $x = 2$

2. (6.00)
From first solution $\text{k} = 1.6, \text{R} = 60, \text{M} = 0.2$
$\text{K} = \frac{l}{\text{AR}}$
$\implies \frac{l}{\text{A}} = 60 \times 1.4$
For second solution, $\text{R} = 320, \frac{l}{\text{A}} = 60 \times 1.4$
$\implies$ k=0.3 and molar conductivity $= \frac{k}{1000m}$
$= 6 \times 10^{-4}\ \text{Sm}^2\text{mol}^{-1}$

3. (6.54)
   $2MnO_4^- + 6H^+ + 5H_2O_2 \rightarrow 2Mn^{2+} + 8H_2O + 5O_2$
   From given reaction,
   Molarity of $KMnO_4$=0.07=no. of moles of $KMnO_4$/volume in litres
   No.of moles of $KMnO_4$=7×10^{-3}
   Moles of H_2O_2=$\frac{5 \times \text{moles of } KMnO_4}{2}$
   Molarity of H_2O_2=0.584
   Volume strength of H_2O_2=11.2×Molarity=6.54

4. (6.00)
   C=H_3BO_3 anhydride of C=B_2O_3 $m = 1, n = 5, m + n = 6$

5. (4.00)
   This is a reaction between acetaldehyde and excess formaldehyde to give pentaerithritol so 3 aldols are possible $m = 3, n = 1, m+n = 4$

Mathematics

Single Option Correct

1. (Question Cancelled)
2. (D)

$$I = \int_0^\infty \frac{x^2}{(x^2+1)(x^2+4)(x^2+9)}dx$$

Using method of partial fractions where $x^2 = t$

$$\frac{t}{(t+1)(t+4)(t+9)} = \frac{A}{t+1} + \frac{B}{t+4} + \frac{C}{t+9}$$

Solving these equations, we get,

$$A = \frac{-1}{24}$$
$$B = \frac{4}{15}$$
$$C = \frac{-9}{40}$$

Plugging in these values and using

$$\int_0^\infty \frac{dx}{x^2+a^2} = \frac{\pi}{2a}$$

Then the value of

$$I = \frac{\pi}{120}$$

3. (D)

$$K = \int_0^{\frac{\pi}{4}} \sqrt{\sec^2 x + 1}dx$$
$$= \int_0^{\frac{\pi}{4}} \frac{\sec^2 x + 1}{\sqrt{\sec^2 x + 1}}$$
$$= \underbrace{\int_0^{\frac{\pi}{4}} \frac{\sec^2 x}{\sqrt{\sec^2 x + 1}}}_{I_1} + \underbrace{\int_0^{\frac{\pi}{4}} \frac{1}{\sqrt{\sec^2 x + 1}}}_{I_2}$$

I_1 can be solved using the substitution $t = \tan x$ whereas I_2 can be solved using the substitution $t = \sin x$

$$I_1 = \int_0^1 \frac{dt}{\sqrt{t^2+2}}$$
$$= \log\left(\frac{1+\sqrt{3}}{\sqrt{2}}\right)$$

$$I_2 = \int_0^{\frac{1}{\sqrt{2}}} \frac{dt}{\sqrt{2-t^2}}$$
$$= \arcsin\frac{1}{2}$$
$$= \frac{\pi}{6}$$

Then we have,

$$I = \log\left(\frac{1+\sqrt{3}}{\sqrt{2}}\right) + \frac{\pi}{6}$$

4. (D)
Let $c = \int_1^2 y\ dx$. Then we get the L.D.E.,

$$xy' - y = cx$$
$$y' - \frac{y}{x} = c$$

Then Integrating Factor,

$$I.F. = \frac{1}{x}$$

Solving this differential equation, we get,

$$\frac{y}{x} = c\ln x + D$$

At $x = 1,\ y = 2$ thus we have $D = 2$.
Using the relation $c = \int_1^2 y\ dx$, we get the following equation

$$c\int_1^2 x\ln x\ dx + \int_1^2 2x\ dx = c$$
$$c\left(2\ln 2 - \frac{3}{4}\right) + 3 = c$$
$$c = \frac{12}{7 - 8\ln 2}$$

5. (B)
Tangent of a parabola

$$y = mx + \frac{a}{m}$$
$$= mx + \frac{1}{m}$$

Tangent of an ellipse

$$y = mx + \sqrt{a^2m^2 + b^2}$$
$$= mx + \sqrt{4m^2 + 3}$$

Equating these tangents,

$$\frac{1}{m} = \sqrt{4m^2 + 3}$$
$$m = \pm\frac{1}{2}$$

Common tangents are,

$$y = \frac{x}{2} + 2$$

and

$$y = \frac{-x}{2} - 2$$

Point A is (-4,0), for B and C, $x = -2$, $y = \frac{-2}{2} + 2 = 1$ and $y = \frac{2}{2} - 2 = -1$
Hence, B(-2,1) and C(-2,-1)

$$Area = \frac{1}{2} \times \text{Base} \times \text{Height}$$
$$= 2 \text{ sq.units}$$

6. (Question Cancelled)

7. (A)
Consider the parabola $y = Ax^2 + Bx + C$
Its vertex is given by $\left(\frac{-B}{2A}, \frac{-(B^2-4AC)}{4A}\right)$
For the given parabola,

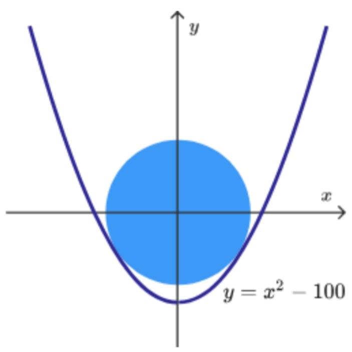
$$A = \frac{a^3}{3}$$
$$B = \frac{a^2}{2}$$
$$C = -2a$$

So, the vertex is,

$$\left(\frac{-\frac{a^2}{2}}{2\frac{a^3}{3}}, \frac{-\left(\frac{a^4}{4} - 4\left(\frac{a^3}{3}\right)(-2a)\right)}{4\frac{a^3}{3}}\right) = \left(\frac{-3}{4a}, \frac{-35a}{16}\right)$$

Therefore, $xy = \frac{105}{64}$

8. (D)



From the figure, it is obvious that the graph is symmetric about the Y$-axis$.
Hence the two points on the circle which touch the parabola must have the same y coordinate.

Let equation of the circle be $x^2 + y^2 = a^2$, where a is the radius of the circle.
Then the points of intersection are obtained by solving the equation of the circle and the equation of the parabola.
The equations:

$$x^2 + y^2 = a^2$$
$$y = x^2 - 100$$

Substituting the value of x^2 from first equation in second equation, we get:

$$y = a^2 - y^2 - 100$$

which rearranges to:

$$y^2 + y + 100 - a^2 = 0$$

Clearly since the value of y coordinates of the two points must be same, this equation should have only one solution, i.e, the discriminant of this quadratic equation must be zero. So,

$$1^2 - 4(1)(100 - a^2) = 0$$

Hence

$$a^2 = \frac{399}{4}$$
$$\text{Area} = \frac{399}{4}\pi$$

Clearly $a + b = 399 + 4 = 403$

9. (A)

$$f'(x) = e^{x(1-x)} + xe^{x(1-x)} \cdot (1 - 2x)$$
$$= e^{x(1-x)} \left(1 + x(1 - 2x)\right)$$
$$= e^{x(1-x)} \left(-2x^2 + x + 1\right)$$

$e^{x(1-x)}$ is always positive
$(-2x^2 + x + 1)$ is positive in the interval $\left[-\frac{1}{2}, 1\right]$ and negative elsewhere
Therefore,

$$f(x) \text{ is increasing on } \left[-\frac{1}{2}, 1\right]$$

10. (B)
Let $F(x) = f(x) - 2g(x)$
Applyting Rolle's Theorem to $F(x)$

$$F(0) = 0$$

$$F(1) = f(1) - 2g(1)$$
$$0 = 6 - 2g(1)$$
$$g(1) = 3$$

TENSORS

11. (A)
We know that a straight line passing through (x_1, y_1) and (x_2, y_2) is given by,

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

Given (x, y) lies on the line $x + y = 2$. So the minimum value of $\sqrt{x^2 + y^2}$ will be the perpendicular distance of the line from origin.

$$\text{Minimum value of } \sqrt{x^2 + y^2} = \sqrt{2}$$

12. (C)

$$|P| = 2\alpha - 6$$
$$P = \text{adj}A$$

$$|P| = |\text{adj}A|$$
$$= (|A|)^2$$
$$= 16$$

$$2\alpha = 22$$
$$\alpha = 11$$

Max value of $x + 2y + 3z$
Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\vec{a} \cdot \vec{b} = x + 2y + 3z \leq |a||b|$$

$$x + 2y + 3z \leq \sqrt{14}$$

13. (D)

$$AA^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$
$$\Rightarrow A^T = A^{-1}$$

Hence, Trace $= \frac{5}{3}$

14. (A)
First, evaluate numerator,

$$(1 + x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \ldots {}^nC_n x^n$$

Differentiating both sides,

$$n(1 + x)^{n-1} = 1 \cdot {}^nC_1 + 2 \cdot {}^nC_2 x + 3 \cdot {}^nC_3 x^2 + \ldots + n \cdot {}^nC_n x^{n-1}$$

Multiplying both sides by x

$$nx(1 + x)^{n-1} = 1 \cdot {}^nC_1 x + 2 \cdot {}^nC_2 x^2 + 3 \cdot {}^nC_3 x^3 + \ldots + n \cdot {}^nC_n x^n$$

Differentiating again,

$n((1 + x)^{n-1} + (n - 1)x(1 + x)^{n-2}) =$
$1^2 \cdot {}^nC_1 + 2^2 \cdot {}^nC_2 x + 3^2 \cdot {}^nC_3 x^2$
$+ \ldots + n^2 \cdot {}^nC_n x^{n-1}$

Putting in $x = 1$

$$n(2^{n-1} + (n - 1)2^{n-2}) = \text{numerator}$$
$$= 2^{n-2}n(2 + n - 1)$$
$$= 2^{n-2}n(n + 1)$$

Now, Denominator,

$$2^n \sum_{r=0}^{n} r = 2^n \frac{n(n + 1)}{2}$$
$$= 2^{n-1}n(n + 1)$$

Therefore, the limit is,

$$\lim_{n\to\infty} \frac{2^{n-2}n(n + 1)}{2^{n-1}n(n + 1)} = \frac{1}{2}$$

15. (C)
$x_1 - 1 = 0$
Let $X_1 = x_1 - 1$. This implies $X_1 \geq 0$
Similarly, $X_2 = x_2 - 2$ and $X_3 = x_3 - 3$ and $X_2 \& X_3 \geq 0$
Therefore,
$x_1 + x_2 + x_3 = n$ can be written as, $X_1 + X_2 + X_3 = n - 6$, where $X_1, X_2, X_3 \geq 0$
Number of solutions of (x_1, x_2, x_3) will be the same as that of (X_1, X_2, X_3)
Therefore, Number of solutions (from multinomial theorem)

$$= {}^{n-6+3-1}C_{3-1} = {}^{n-4}C_2$$

16. (C)

$$\text{Arithmetic Mean} \geq \text{Geometric Mean}$$
$$\frac{4^{\sin^2 x} + 4^{\cos^2 x}}{2} \geq \sqrt{4^{\sin^2 x} \cdot 4^{\cos^2 x}}$$
$$\geq \sqrt{4^{\sin^2 x + \cos^2 x}}$$
$$\geq 2$$
$$4^{\sin^2 x} + 4^{\cos^2 x} \geq 4$$

17. (A)
18. (D)
19. (A)
Let the other 2 numbers be x_1, x_2

$$\text{Mean} = \frac{x_1 + x_2 + 2 + 3 + 6}{5} = 5.8$$
$$x_1 + x_2 = 18$$

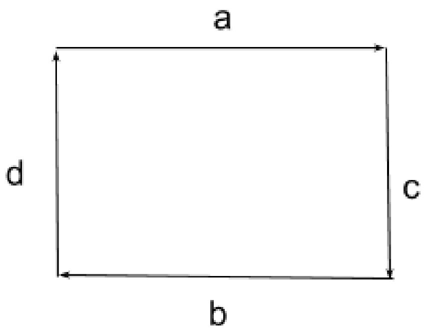
$$\text{Variance} = \frac{1}{5}\sum_{i=1}^{5}(x_i - \bar{x})^2$$
$$= \frac{(x_1-5.8)^2}{5} + \frac{(x_2-5.8)^2}{5} + \frac{(3.8)^2}{5} + \frac{(2.8)^2}{5} + \frac{(0.2)^2}{5}$$
$$= 10.16$$

$$(x_1+x_2)^2 = 170$$
$$x_1^2 + x_2^2 + 2x_1x_2 = 170$$
$$18^2 + 2x_1x_2 = 170$$
$$2x_1x_2 = 154$$
$$(x_1-x_2)^2 = x_1^2 + x_2^2 - 2x_1x_2$$
$$= 170 - 154$$
$$= 16$$
$$x_1 - x_2 = \pm 4$$

Let $x_1 > x_2$
Then,

$$x_1 - x_2 = 4$$
$$x_1 + x_2 = 18$$
$$\Rightarrow x_1 = 11 \text{ and } x_2 = 7$$

20. (A)



$$\vec{a} + \vec{b} + \vec{c} + \vec{d} = \vec{0}$$
$$\vec{a}\cdot\vec{d} + \vec{b}\cdot\vec{d} + \vec{c}\cdot\vec{d} + \vec{d}\cdot\vec{d} = \vec{0} \qquad (1)$$

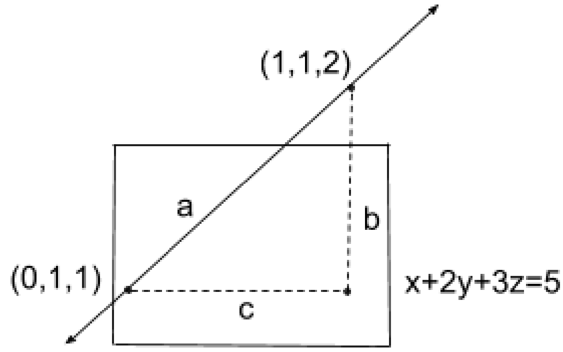
$$\left(\vec{a}+\vec{b}\right)\cdot\left(\vec{c}+\vec{d}\right) = \vec{0}$$
$$\vec{a}\cdot\vec{c} + \vec{a}\cdot\vec{d} + \vec{b}\cdot\vec{c} + \vec{b}\cdot\vec{d} = \vec{0} \qquad (2)$$

Comparing eqn (1) and (2)

$$\vec{c}\cdot\vec{d} + \vec{d}\cdot\vec{d} = \vec{a}\cdot\vec{c} + \vec{b}\cdot\vec{c}$$
$$\vec{d}\cdot\left(\vec{c}+\vec{d}\right) = \vec{c}\cdot\left(\vec{a}+\vec{b}\right)$$
$$-\vec{d}\cdot\left(\vec{a}+\vec{b}\right) = \vec{c}\cdot\left(\vec{a}+\vec{b}\right)$$
$$\therefore \vec{c} = -\vec{d}$$
$$\therefore \vec{a} = -\vec{b}$$

Numerical Answer

1. (0.86)



The foot of the perpendicular from $(1,1,2)$ to the plane is $(\frac{5}{7}, \frac{3}{7}, \frac{8}{7})$
From figure,

$$b^2 + c^2 = a^2$$
$$c^2 = a^2 - b^2$$
$$= \left(\frac{5}{7}\right)^2 + \left(\frac{4}{7}\right)^2 + \left(\frac{1}{7}\right)^2$$
$$= \frac{42}{49}$$
$$= 0.86$$

2. (11.25)
Evaluate the integral by susbtituting $t = 2x$,

$$K = \frac{1}{64}\int_0^\infty t^6 e^{-t} dt$$

Consider $I_n = \int_0^\infty t^6 e^{-t} dt$
Using integration by parts, we get,

$$I_n = \left[t^n(-e^{-t})\right]_0^\infty - \int_0^\infty nt^{n-1}(-e^{-t})dt$$

Thus, we get the identity,$I_n = nI_{n-1}$
Therefore, $I_n = n!$
Hence we get, $K = 11.25$

3. (5.00)
$\mathbb{P}(A_i) = ki$, where k is the proportionality constant.
However, it is known to us that $\sum_{i=0}^{n}\mathbb{P}(A_i) = 1$
So,

$$k\sum i = 1$$
$$\Rightarrow \frac{kn(n+1)}{2} = 1$$
$$\Rightarrow k = \frac{2}{n(n+1)}$$

$$\therefore \mathbb{P}(A_i) = \frac{2i}{n(n+1)}$$

Probability of white from bag A_i,

$$\mathbb{P}(\text{White}|A_i) = \frac{i}{n-i+i} = \frac{i}{n}$$

$\therefore$

$$\begin{aligned}\mathbb{P}(\text{White}) &= \sum_{i=0}^{n} \mathbb{P}(\text{White}|A_i) \cdot \mathbb{P}(A_i) \\ &= \sum_{i=0}^{n} ki\frac{i}{n} \\ &= \frac{k}{n}\sum_{i=0}^{n} i^2 \\ &= \frac{k}{n}\frac{n(n+1)(2n+1)}{6} \\ &= \frac{2}{n(n+1)}\frac{(n+1)(2n+1)}{6} \\ &= \frac{2n+1}{3n}\end{aligned}$$

$\therefore$

$$\lim_{n\to\infty} \mathbb{P}(\text{White}) = \lim_{n\to\infty} \frac{2n+1}{3n} = \frac{2}{3} = \frac{a}{b}$$

$\therefore a + b = 5$

4. (2.00)
$f(x)$

$$\begin{aligned}2 - x &> 0 \\ x &< 2\end{aligned}$$

$$\begin{aligned}\ln(2-x) &\neq 1 \\ 2 - x &\neq 1 \\ x &\neq 1\end{aligned}$$

also,

$$\begin{aligned}x &\geq 0 \\ \therefore \text{A} &= [0, 2) - \{1\}\end{aligned}$$

$g(x)$

$$\begin{aligned}x^2 - 4 &> 0 \\ x^2 &> 4 \\ x < -2 &\text{ or } x > 2 \\ x + 3 &\geq 0 \\ x &\geq -3 \\ \text{B} &= [-3, -2) \cup (2, \infty)\end{aligned}$$

$h(x)$

$$\begin{aligned}x + |x| &\neq 0 \\ \implies x &> 0 \\ \text{C} &= (0, \infty)\end{aligned}$$

$$\begin{aligned}(\text{A}' - \text{B}) \cap \text{C} &= \{1, 2\} \\ \implies n((\text{A}' - \text{B}) \cap \text{C}) &= 2\end{aligned}$$

5. (4.00)
$Z = iZ_1$
Let $Z_1 = \cos\theta + i\sin\theta$

$$\begin{aligned}Z &= e^{i(\cos\theta + i\sin\theta)} \\ &= e^{i\cos\theta - \sin\theta} \\ |Z| &= e^{-\sin\theta} \\ \therefore e^{-1} &\leq Z \leq e^{1} \\ \implies 0.37 &\leq Z \leq 2.72\end{aligned}$$