Tensors 2020

JEE Advanced

December 22, 2019

Answer Key

Multiple Option Correct

Physics	Chemistry	Mathematics
1.(A,D)	1.(A,C,D)	1.(A,B,D)
2.(B,D)	2.(B)	2.(D)
3.(C)	3.(A,D)	3.(A,C)
4.(C)	4.(B,C)	4.(B,C)
5.(D)	5.(B,C,D)	5.(A,B)
6.(A,C)	6.(A,B,D)	6.(B,D)
7.(A,D)	7.(B,D)	7.(A,B)
8.(D)	8.(B,D)	8.(A,C)

Numerical Answer

Physics	Chemistry	Mathematics
1.(1.00)	1.(0.00)	1.(0.48)
2.(1.00)	2.(3.33)	2.(8.00)
3.(14-15/0.14)	3.(3.00)	3.(1.96)
4.(0.30)	4.(7.00)	4.(16.63)
5.(0.40)	5.(32.54)	5.(3.00)
6.(0.63-0.65)	6.(78.00)	6.(1.00)

Paragraph Comprehension

Physics	Chemistry	Mathematics
1a.(B)	1a.(D)	1a.(B)
1b.(A)	1b.(D)	1b.(A)
2a.(B)	2a.(A)	2a.(B)
2b.(D)	2b.(B)	2b.(B)

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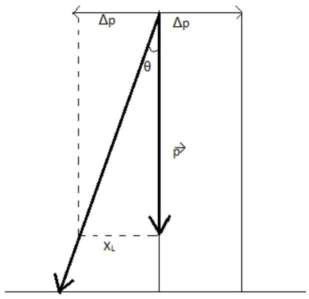
Solutions

Physics

Multiple Options Correct

1. (A,D)
Net force = weight - buoyant force
Weight = mg = 50N
Buoyant force = $\rho v g = \dfrac{1000 \times 4\pi r^3 \times 10}{3} =$ 41.866N
therefore net force =8.134N
Since the buoyant force will be acting through the geometrical centre, the body will be rotating about its centre of mass.
Thus torque = magnitude of buoyant force x distance between geometrical centre and centre of mass.
Torque = 41.866 x 0.075 = 3.14Nm

2. (B,D)



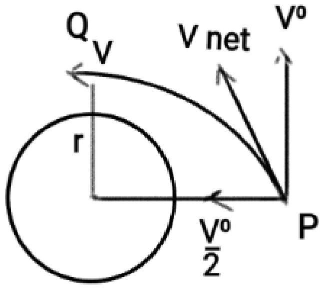
As the width of the slit is decreased , the beam gets narrow initially. But as the width is further decreased, the Heisenberg's uncertainty principle comes into effect.
$\triangle x = 10^{-6}$
$\lambda = \dfrac{h}{p}$ de-broglies wavelength.

$$p = \frac{h}{\lambda} \approx 10^-27$$
$$\triangle x \times \triangle p = \frac{h}{2\pi}$$

$$\triangle p = \frac{6.63 \times 10^{-34}}{2 \times 3.14 \times 10^-4} \approx 10^{-30}$$
$$\theta = \tan\theta = \frac{\triangle p}{p} = 10^{-3}$$
$$x_L = \theta L = 10^{-1}$$

There for the length of beam is approximately 2×10^{-1}m

3. (C)



The orbital velocity of the satelite is $V_0 = \left[\dfrac{GM}{a}\right]^{\frac{1}{2}}$ ....(1)
From conservation of angular momentum at P and Q;

$$mav_0 = mvr$$
$$v = \frac{av_0}{r}....(2)$$

Energy conservation at P and Q:

$$\frac{m}{2}(v_0^2 + \frac{v_0^2}{4}) - \frac{GMm}{a} = \frac{mv^2}{2} - \frac{GMm}{r}$$
$$\frac{5V_0^2}{8} - \frac{GM}{a} = \frac{v^2}{2} - \frac{GM}{r}.....(3)$$

Solving (1)(2)(3)
we get $r = 2a, \dfrac{2a}{3}$
Thus $r_{min} = \dfrac{2a}{3}$

4. (C)

When shell A is earthed we know that the potential at all points on shell A is zero .But the potential is not zero at the centre of the shell A as a charge q is contained in it.But the potential on shell A due to charge q inside and the induced charges on shell A is zero. So the inner charges have no role in the change in the charge distribution on the outer surface of shell A to make the potential on the surface zero.
So we can consider the inner charges not to be present and then equate the potential at the centre of the shell A to be zero. If the final charge on shell A is x.

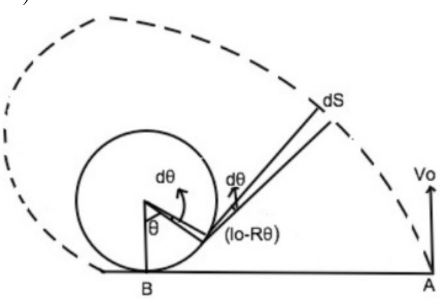
Then

$$\frac{kx}{R} + \frac{3kq}{2R} + \frac{k(2Q+q)}{5R} = 0$$
$$x = \frac{-17q}{10} - \frac{2Q}{5}$$

So the magnitude of the charge flown through the switch is

$$F(Q,q) = Q + q + \frac{17q}{10} + \frac{2Q}{5}$$
$$F(5,10) = 34$$

5. (D)



Here the tension is always perpendicular to velocity
Work done by tension is 0
Velocity of the disk v_0 is unchanged throughout

Time taken; $t = \frac{s}{v_0}$
At an arbitrary position(see fig)

$$ds = (l_0 - r\theta)d\theta$$
$$s = \int_{\theta=0}^{\theta=\frac{l_0}{R}} (l_0 - R\theta)d\theta$$
$$= \frac{l_0^2}{R} - R \times \frac{l_0^2}{2R^2}$$
$$= \frac{l_0^2}{2R}$$

Therefore time taken

$$t = \frac{s}{v}$$
$$= \frac{l_0^2}{2RV_0}$$

6. (A,C)
We know for a SHM Total energy is a constant.
KE + PE = constant
Total kinetic energy include rotational and translational kinetic energies.
Rotational kinetic energy= $\frac{I\omega^2}{2}$
Translational kinetic energy = $\frac{mv^2}{2}$
Potential energy =$\frac{kx^2}{2}$
We can replace x as $R\theta$

$P.E = \frac{kR^2\theta^2}{2}$
Sum of all these is a constant.
R.K.E + T.K.E + P.E =$\frac{mv^2}{2} + \frac{kx^2}{2} + \frac{I\omega^2}{2} =$ *constant*
Differentiating the above equation we get,

$$\alpha(mR^2 + I) + kR^2\theta = 0$$
$$\alpha + \frac{kR^2\theta}{mR^2 + I} = 0$$

Comparing with SHM equation,

$$\omega = \sqrt{\frac{kR^2\theta}{mR^2 + I}}$$
$$I = \frac{MR^2}{2}$$
$$\omega = \frac{k}{m + \frac{M}{2}}$$

So the oscillations are SHM ,no matter what so (A) is True, (B) is False

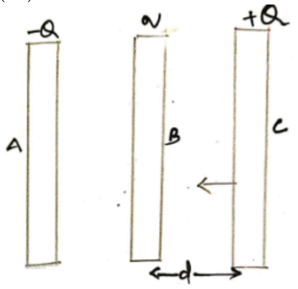
put m= M = 0.2 kg. k = 1.2 N/m we will get frequency= $\frac{1}{\pi}$; (C) is true
Time period is independent of Radius R ;(D) is false

7. (A,D)
λ is the de-broglie wavelength of electron emitted from the metal.
Its energy is $2.55eV - 2.14eV = 0.41eV$.

$$\lambda = \frac{h}{\sqrt{2mKE}} = \frac{h}{p}$$
$$\frac{h}{mv} = \frac{h}{\sqrt{2m \times \frac{mv^2}{2}}}$$
$$= 1.918 \times 10^{-9} m$$

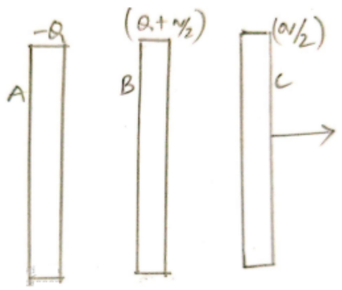
The energy levels in a hydrogen atom are $E = \frac{-13.6}{n^2}$
$n_1 - n_2 = 2$

8. (D)



At any instant the force on plate c = $\frac{(Q-q)Q}{2S\epsilon_0}$
Work done = $\frac{(Q-q)Qd}{2S\epsilon_0}$

After the collision charge on plate A,B,C



work done= $\frac{(Q + \frac{q}{2} - Q)\frac{q}{2}d}{2S\epsilon_0} = \frac{(\frac{q}{2})^2 d}{2S\epsilon_0}$

total work done= $\frac{(Q-q)Qd}{2S\epsilon_0} + \frac{(\frac{q}{2})^2 d}{2S\epsilon_0} = \frac{(Q - \frac{q}{2})^2 d}{2S\epsilon_0}$

$$v = Q - \frac{q}{2}\sqrt{\frac{d}{mS\epsilon_0}}$$
$$= \frac{3}{2} - \frac{1}{2}\sqrt{\frac{d}{mS\epsilon_0}} = \sqrt{\frac{d}{mS\epsilon_0}}$$

Numerical Answer

1. (1)
parametric form is $(t^2, 2t)$
Now consider a small element $\vec{dl}$ at some t
As $x = t^2, y = 2t$;
$dx = 2tdt, dy = 2dt$

$$\vec{dl} = dx\hat{i} + dy\hat{j}$$
$$= 2dt(t\hat{i} + \hat{j})$$

Applying Biot-Savart Law at the focus of the parabola (1,0)

$$\vec{dB} = \frac{\mu i}{4\pi}\frac{\vec{dl} \times \vec{r}}{r^3}$$

$$r = (1 - t^2)\hat{i} + 2t\hat{j}$$
$$|\vec{r}| = 1 + t^2$$
$$\vec{dl} \times \vec{r} = 2dt(3t^2 - 1)\hat{k}$$
$$\vec{dB} = \left(\frac{\mu i}{4\pi}\right)\frac{2(3t^2 - 1)dt}{(1 + t^2)^3}\hat{k}$$
$$\vec{dB} = \left(\frac{\mu i}{2\pi}\right)\frac{(3t^2 - 1)dt}{(1 + t^2)^3}\hat{k}$$

$$B = \int dB = \int_{-1}^{1}\left(\frac{\mu i}{2\pi}\right)\frac{(3t^2 - 1)dt}{(1 + t^2)^3}$$
$$= \left(\frac{\mu i}{2\pi}\right)\left[\frac{-t}{(t^2 + 1)^2}\right]_{-1}^{1}$$
$$= -\frac{\mu i}{4\pi}$$
$$|B| = 10^{-7}T$$

2. (1)
Buoyant force is defined as the vertical upward force applied by a liquid on a body due to pressure difference. Here, to experience buoyant force, the force on top of the big cylinder should be less than that at the base of the big cylinder(the liquid does not apply force at the base of the small

cylinder since there is no liquid beneath it).

ie $\pi(R^2-r^2)(h-h_2)\rho g > \pi R^2[h-(h_1+h_2)]\rho g$
simplifying; $R^2h_1 > r^2(h-h_2)$
$\therefore n = 1$

3. (14-15 / 0.14)
Initially, A is in extreme position when A reaches mean position its velocity will be maximum which is given by ωA, where the angular frequency is 10 rad/s and amplitude is 5m

Velocity at mean position $= \omega A = 10 \times 5 = 50$
For the elastic collision,
relative velocity of separation = relative velocity of approach
$V_B - V_A = U_A - U_B$
$V_B - V_A = 50$ – – – – – – – – – – – – – – – – – – (1)
Using the law of conservation of momentum
$M_AV_A + M_BV_B = M_AU_A + M_BU_B$
$2V_B + V_A = 50$ – – – – – – – – – – – – – – – – –(2)
Solving (1) and (2)

$$V_B = \frac{100}{3} \quad V_A = \frac{-100}{6}$$

Here we can see V_A is negative which means it goes back to the same extreme it started and comes back.so it clearly takes 3T/4 to get to the other extreme after the collision . After the collision amplitude of SHM changes but time period remains constant.

Time period ,T $= \frac{2\pi}{\omega} = \frac{\pi}{5}$
So time taken to reach other extreme after collision $= \frac{3T}{4} = \frac{3\pi}{20}s$
New amplitude $= \frac{V_A}{\omega} = \frac{10}{6}$
In this for $\frac{3\pi}{20}$ second B travels with velocity $\frac{100}{3}$.

Distance between A and B when A is at extreme $= \left(\frac{3\pi}{20} \times \frac{100}{3}\right) - (\frac{10}{6}) = \frac{295}{21} =$ 14.05m

4. (0.30)

$$\frac{R_1}{R/-2} = \frac{3}{2}$$

$$\frac{R_1^3}{R_2^3} = \frac{27}{8}$$

The density of the nucleus is same for all atoms.

$$\frac{m_1}{\left(\frac{4\pi R_1^3}{3}\right)} = \frac{m_2}{\left(\frac{4\pi R_2^3}{3}\right)}$$

$$\frac{m_1}{m_2} = \frac{R_1^3}{R_2^3} = \frac{27}{8}$$

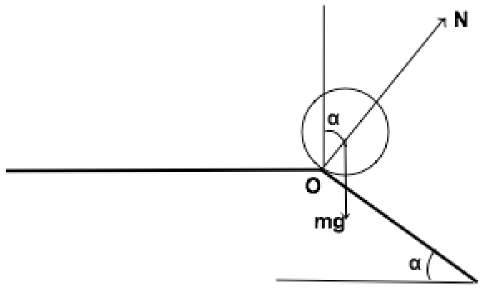
As number of neutrons equals to number of protons

$$\frac{m_1}{m_2} = \frac{2z_1}{2z_2} = \frac{z_1}{z_2}$$

The radius of K shell is inversely proportional to z.

$$\frac{R_1}{R_2} = \frac{z_2}{z_1} = \frac{8}{27}$$

5. (0.40)



Since the cylinder moves without sliding, the centre of the cylinder rotates about the point O, while passing through common edge of plane, ie, O become IAOR
If at any instant the velocity of COM is V_1 when angle is β;

$$\frac{mV_1^2}{R} = mg\cos\beta - N$$

$$V_1^2 = gR\cos\beta - \frac{NR}{m}\text{....................(1)}$$

Conservation of energy

$$\frac{I_0V_1^2}{2R^2} - \frac{I_0V_0^2}{2R^2} = mgR[1-\cos\beta]$$

Where $I_0 = [\frac{mR^2}{2} + mR^2] = \frac{3mr^2}{2}$

$$V_1^2 = V_0^2 + \frac{4gR[1-\cos\beta]}{3}\text{.................(2)}$$

From (1) and (2)

$$V_0^2 = \frac{gR[7\cos\beta - 4]}{3} - \frac{NR}{m}$$

For $\beta = \alpha$

$$V_0^2 = \frac{gR[7\cos\alpha - 4]}{3} - \frac{NR}{m}$$

No jumping occurs if $N > 0$
V_0 must be less than

$$V_{max} = \sqrt{\frac{gr[7\cos\alpha - 4]}{3}}$$

Substituting all the values to get 0.4

6. (0.63-0.65)
Avg velocity$=\frac{\text{Total displacement}}{\text{Total time}} = \frac{\text{Range}}{\text{Time taken}}$

$$V_{avg} = \frac{\frac{(U^2)2\sin\theta\cos\theta}{g} + \frac{\left(\frac{U}{\alpha}\right)^2 2\sin\theta\cos\theta}{g} + ...}{\frac{2U\sin\theta}{g} + \frac{\frac{2U}{\alpha}\sin\theta}{g} + ...}$$

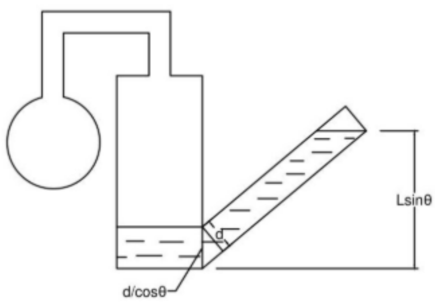
$$= \frac{\frac{U\cos\theta}{1-(\frac{1}{\alpha})^2}}{\left(\frac{1}{1-\frac{1}{\alpha}}\right)}$$

$$= U\cos\theta\left(\frac{\alpha}{\alpha+1}\right)$$

So ratio$=\frac{U\cos 60^\circ\left(\frac{\alpha}{\alpha+1}\right)}{U\cos 30^\circ\left(\frac{\alpha}{\alpha+1}\right)} = \frac{\sqrt{3}}{2} = 0.85$

Paragraph Comprehension

1.a (B)



Pressure is measured by finding the height difference between both sides. The maximum height difference that can be made is as shown in the figure.
Maximum height difference,
$h = L\sin\theta - \frac{d}{\cos\theta}$

Maximum pressure difference,
$P - P_a = (L\sin\theta - \frac{d}{\cos\theta})\rho g$
Therefore maximum pressure that can be measured $= (L\sin\theta - \frac{d}{\cos\theta})\rho g + P_a$

1.b (A)
Volume of liquid lowered in the tank = Volume of liquid increased in the slanting tube
Let the liquid level in the slanting tube increase by a slanting length l. Then,

$$D^2h = \frac{\pi l d^2}{4}$$
$$L = \frac{4D^2h}{\pi d^2}$$

Total height difference $= h + l\sin\theta$
$= h + \frac{4D^2h\sin\theta}{\pi d^2}$

Pressure of the gas $= (h + \frac{4D^2h\sin\theta}{\pi d^2})\rho g + P_a$

2.a (B)

$$m(r\ddot{\theta} + 2\dot{r}\dot{\theta}) = eB\dot{r}$$
$$\frac{d}{dt}(r^2\dot{\theta}) = \frac{eBr}{m}\dot{r}$$
$$\frac{d}{dt}(r^2\dot{\theta}) = \frac{eB}{2m}\frac{d}{dt}(r^2)$$
$$\dot{\theta} = \frac{eB}{2m}\left(1 - \frac{a^2}{r^2}\right) \qquad \left\{\frac{d\theta}{dt} = 0 \text{ for } r = a\right\}$$

2.b (D)

$$v^2 = \frac{2eV}{m}$$
$$V = r\theta$$

Substitute the values to find V

$$V = \frac{9e}{8m}$$

Chemistry

Multiple Option Correct

1. (A,C,D)
   Acidified orange soution = $K_2Cr_2O_7$
   Lunar caustic =$AgNO_3$
   Hypo=$Na_2S_2O_3$
   Group III A compound=Al
   Caustic soda=NaOH
   Light blue colour solution=$CuSO_4$
   Ammonia water=NH_4OH
   Prussiate of potash=$K_4[Fe(CN)_6]$

2. (B)
   Fact

3. (A,D)
   A. Order is I > Cl > Br
   D. $I_2+5Cl_2+6H_2O \rightarrow 2HIO_3+10HCl$

4. (B,C)
   $Q=\frac{\left[A^{n+}\right]^2}{\left[B^{2n+}\right]} = 4$

$$\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$$
$$\Delta H^\circ = 2\Delta G^\circ$$

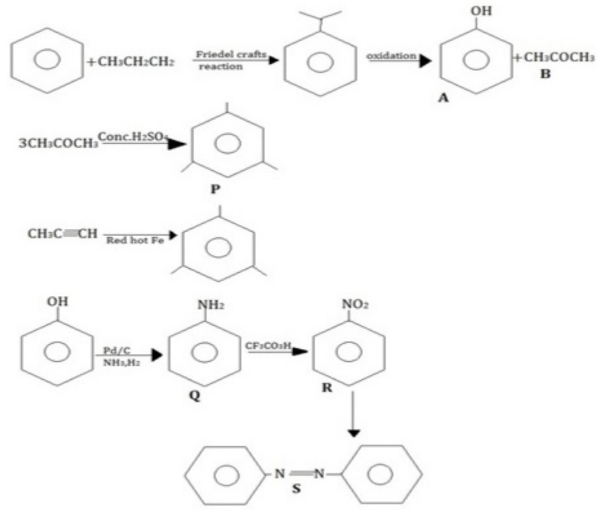
$$\Delta G = \Delta G^\circ + RT\ln Q$$
$$\Delta G = 0$$

$$\Delta S = -R\ln K$$
$$= -11.62 J/Kmol$$

$$\Delta G^\circ = -nFE^\circ_{cell}$$
$$E^\circ_{cell} = \frac{RT}{nF} \times \ln K$$
$$= 0.018V$$

5. (B,C,D)
   B. isoelectric point $=\frac{pKa_2 + pKa_3}{2}$
   C. Fact
   D. Fact

6. (A,B,D)



7. (B,D)
   B. In Ziese salt $[PtCl_3(C_2H_4)]$ due to back bonding from Pt to C_2H_4 bond order decrease and hence C-C bond length lies between that of single bond(0.153 nm) and double bond(0.134 nm) D. Fact

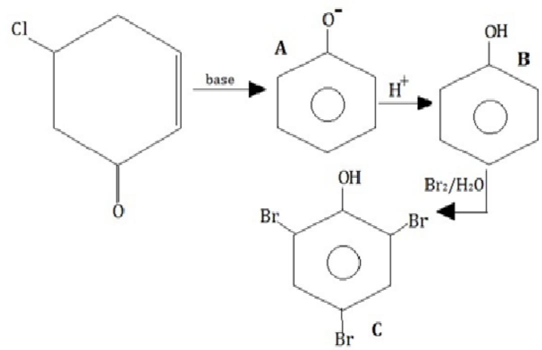
8. (B,D)
   Fact

Numerical Answer

1. (0.00)
   Compound is $[Cr(H_2O)_6]Cl_3$

2. (3.33)
   $a = 6, b = 2, c = 4, d = 3, e = 2, f = 2$

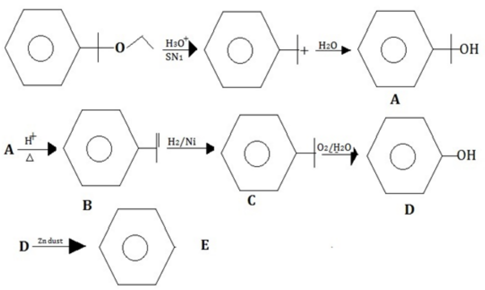
3. (3.00)



4. (7.00)
   Correct statements :1,2,4
   Facts

5. (32.54)
A=H_3PO_4, $x = 3$
B=NCl_3, $y = 4$
C=HCl, $z = 18$
D=S_2Cl_2, $w = 6$

6. (78.00)



Paragraph Comprehension

1a. (D)

$$k_b = 1.5 \times 10^{-11}$$
$$k_a = 6.6 \times 10^{-4}$$
$$pk_a = 3.18$$

$HX + H_2O \rightarrow H_3O^{+} + X^{-}$

It is a buffer, $pH = pk_a + \log_{10}\frac{\left[X^{-}\right]}{[HX]}$

$\left[X^{-}\right] = 3$mmoles

Volume of NaOH added $= \frac{3}{0.3}$=10mL

Total Volume = 10+25 = 35mL

1b. (D)

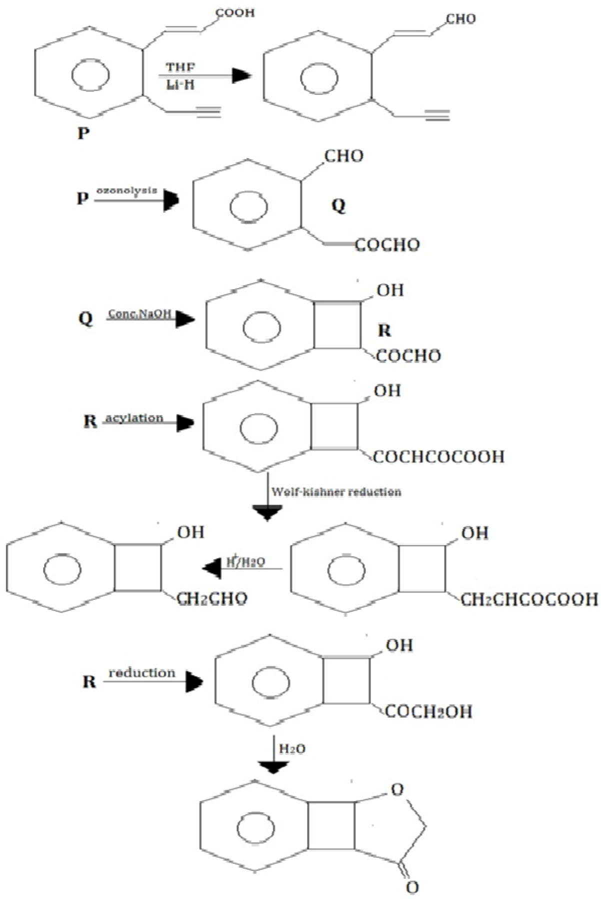
pH of equivalent point $= 7 + \frac{pk_a + \log C}{2} = 8.33$

volume of equivalent point (V) =

$$\frac{\text{Concentation of Acid} \times \text{Volume of Acid}}{\text{Concentation of Base}}$$

V=25mL
V+1=26mL
$pH_2 = 14 - [-\log[OH^{-}(\text{left unreacted})]] = 11.77$
$|pH_2 - pH_1| = 3.44 \approx 3.5$

2.



2a. (A)
2b. (B)

Mathematics

Multiple Option Correct

1. (A,B,D)
Consider the unit circle $|Z|$=1
1,ω,ω^2 are the vertices of the triangle.
Here the given vertex is $-5\omega^2$.
So the other two vertices are $(-5,0)$ and -5ω.

$$\text{Length of the median of the triangle} = 7.5$$
$$\text{Length of the sides of the triangle} = 5\sqrt{3}$$

$$\begin{aligned}\text{Radius of circumcircle} &= \alpha \\ &= 5 \\ \text{Radius of incircle} &= \frac{\text{Radius of circumcircle}}{2} \\ &= 2.5\end{aligned}$$

$$a = -5,\ b = 0,\ c = 2.5,\ d = -\frac{5\sqrt{3}}{2}$$

2. (D)
Take $p = 1$

$$\begin{aligned}\text{we get, } a_2 &= 4 \\ \implies b &= 4\end{aligned}$$

Take $p = 2$

$$\begin{aligned}\text{we get, } a_3 &= 4 \\ \implies c &= 4\end{aligned}$$

Hence the $\triangle$ABC is isosceles
Now, Area $(\triangle)$ = $\sqrt{15}$

$$\begin{aligned}\therefore r_1 &= \frac{\triangle}{s-a} \\ &= \frac{\sqrt{15}}{3} \\ \text{and } r_2 &= \frac{\triangle}{s-b} \\ &= \frac{\sqrt{15}}{1} \\ &= r_3\end{aligned}$$

$$\text{hence, } r_2 = r_3 = 3r$$

3. (A,C)
$A\times A+kI = \begin{bmatrix} a\times a+bc+k & ab+bd \\ ac+cd & d\times d+bc+k\end{bmatrix} = 0$
$k = -(a\times a+bc) = -(d\times d+bc)$
$(a+d)c = (a+d)b = 0$
b, c are not zero since bc is not zero
therefore,

$$\begin{aligned}a+d &= 0 \\ a &= -d \\ k &= -(a*a+bc) \\ &= -(-ad+bc) \\ &= det(A)\end{aligned}$$

.

4. (B,C)

$$\begin{aligned}S(n) &= \Sigma[(^nC_k)^2 + (^nC_{k-1})^2 - 2(^nC_k)(^nC_{k-1})] \\ &= (^{2n}C_n) + (^{2n}C_n) - 2(^{2n}C_{n-1})\end{aligned}$$

When $n = 11$
$S(11) = \frac{(^{22}C_{11})}{6}$
On factorising: $S(11) = 4\times 7\times 13\times 17\times 19$
Thus number of factors excluding itself is 46

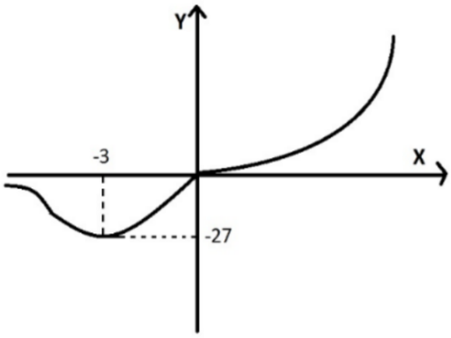
Consider

$$2^{2n} = \Sigma^{2n}C_k$$

Therefore 2^{2n} is the sum of $2n$ terms $(^{2n}C_0 +^{2n}C_{2n})$, $^{2n}C_1$, $^{2n}C_2, \dots,^{2n}C_{2n-1}$ and $^{2n}C_n$ is the biggest among them for $n > 1000$,
we can say that $^{2n}C_n$ is bigger than their average, ie.
$^{2n}C_n > \frac{(2^2n)}{2n}$

After substituting, option C is found to be correct

5. (A,B)
$y = x^3e^{x+3} = -k$
$\frac{dy}{dx} = e^{x+3}(3x^2+x^3)$
$\implies$ $x = 0$,$x = -3$



$\implies$ at $x = -3, y = -27$
for 2 distinct real roots, $k \epsilon (0, 27)$
so prime values of k will be 2, 3, 5, 7, 11, 13, 17, 19, 23
i.e., 9 values

6. (B,D)
1. From the recursion formula we get ,

$$I_k/I_{k-2} = ((k-1)/k)$$
$$I_{k+1}/I_{k-1} = (k/(k+1))$$

Then we have , $\dfrac{(I_k \times I_{k-1})}{(I_{k+1} \times I_{k-2})} = (k^2 - 1)/k^2 < 1$
Then the following are true :

$$I_k \times I_{k-1} < I_{k+1} \times I =_{k-2}$$
$$I_{k+1} \times I_k < I_{k+2} \times I_{k-1}$$

Multiplying these two equations we have :
$(I_k)^2 < I_{k+2} \times I_{k-2}$
Then the following statements are also true :

$$(I_{k+1})^2 < I_{k+3} \times I_{k-1}$$
$$(I_{k-1})^2 < I_{k+1} \times I_{k-3}$$

Multiplying these two inequalities gives :
$I_{k+1} \times I_{k-1} < I_{k+3} \times I_{k-3}$
Similarly we have ,
$I_{k+3} \times I_{k+1} < I_{k+5} \times I_{k-1}$
Multiplying again ,
$(I_{k+1})^2 < I_{k-3} \times I_{k+5}$
Then the following statements are also true :
$(I_k)^2 < I_{k+4} \times I_{k-4}$
$(I_{k+4})^2 < I_k \times I_{k+8}$
Multiplying these two inequalities gives :
$I_{k+4} \times I_k < I_{k-4} \times I_{k+8}$
Also we have
$I_{2k+2} \times I_{2k-2} < I_{2k-6} \times I_{2k+6}$ .... (A) is incorrect
Similarly we have ,
$I_{k+8} \times I_{k+4} < I_{k+12} \times I_k$
Multiplying again,
$(I_{k+4})^2 < I_{k-4} \times I_{k+12}$
Then the following statements are also true :
$(I_{k+12})^2 < I_{k+4} \times I_{k+20}$
$(I_{k+4})^2 < I_{k-4} \times I_{k+12}$
Multiplying these two inequalities gives :
$I_{k+4} \times I_{k+12} < I_{k+20} \times I_{k-4}$
Then we obtain : $I_{2k+4} \times I_{2k-4} < I_{2k+12} \times I_{2k-12}$ .... (B) is correct
On continuous multiplication of the terms,
$\dfrac{4k^2}{(4k^2-1)} = \dfrac{(I_{2k-2} \times I_{2k+1})}{(I_{2k} \times I_{2k-1})}$

We get ,

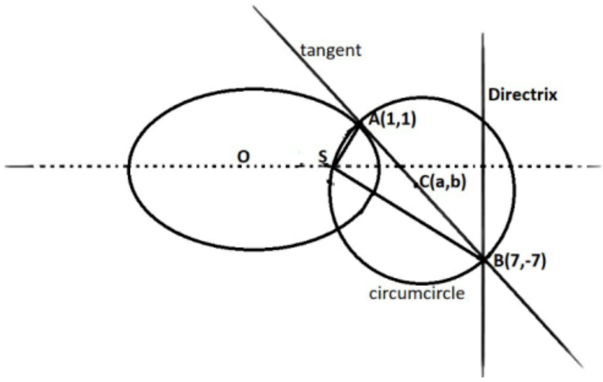
$$P_n = \prod_{k=1}^{k=n} \frac{4k^2}{(4k^2-1)} = \frac{I_0}{I_1} \times \frac{I_{2n+1}}{I_{2n}}$$

We know that , $I_{2n} > I_{2n+1} > I_{2n+2}$
Dividing throughout by I_{2n} ,
$(I_{2n+2}/I_{2n}) < (I_{2n+1}/I_{2n}) < 1$
$(2n+1)/(2n+2) < (I_{2n+1}/I_{2n}) < 1$
As $n \to \infty$ we use the sandwich theorem to get

$\lim_{n\to\infty}(I_{2n+2}/I_{2n}) = 1$
Now since we have
$\lambda = \lim_{n\to\infty} P_n$
$\lambda = \lim_{n\to\infty}(I_{2n+2}/I_{2n}) \times (I_0/I_1)$
Here ,
$I_0 = \pi/2$

$I_1 = 1$
Thus we have ,
$\lambda = \pi/2 = 1.57$ (approx.) .... (D) is correct

7. (A,B)
The center of the ellipse is not at origin. By the property that the portion of the tangent intercepted between the point of contact of tangent (point A) and the directrix subtends a right angle at the corresponding focus.



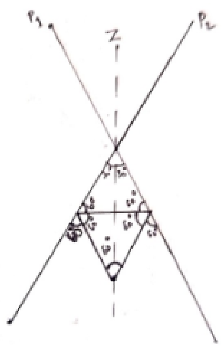
Since ∠ASB is right angle, for circumcircle of △SAB, AB is the diameter, C is the midpoint of AB

Center of circumcircle C is $\left(\frac{1+7}{2}, \frac{1-7}{2}\right) = (4, -3)$

$$a = 4,\ b = -3$$

$$\begin{aligned} a+b &= 1 \\ a-b &= 7 \\ SC^2 &= (radius)^2 \\ &= \frac{AB^2}{4} \\ &= 25 \end{aligned}$$

8. (A,C)
Angle between the planes is 60°. The object is moving parallel to P_2



∴ The locus of the point is the bisector plane in the acute region.
Equation of the bisector plane

$$\begin{aligned} \frac{x-2}{1} &= \pm\frac{x+\sqrt{3}y-4}{2} \\ 2x-4 &= x+\sqrt{3}y-4 \\ \implies x-\sqrt{3}y &= 0 \\ 2x-4 &= x-\sqrt{3}y+4 \\ \implies 3x+\sqrt{3}y-8 &= 0 \end{aligned}$$

Here, $3x+\sqrt{3}y-8=0$ is the bisector plane in the acute angle region

Numerical Answer

1. (0.48)
Let the polynomial be $f(x) = (a_1)x^n + (a_2)x^{n-1} + ...(a_{n-1})x + a_n$

$$\begin{aligned} f(0) = 6 &\implies a_n = 6 \\ f'(0) = -15 &\implies a_{n-1} = -15 \\ f''(0) = 18 &\implies a_{n-2} = 9 \\ f^n(0) = 18 &\implies a_1 = \frac{18}{n} \end{aligned}$$

If 2 and $\frac{2}{3}$ are roots then the polynomial is
$f(x) = (x-1) \times (x-(2/3)) \times h(x)$
$f(x) = (x^2 - \frac{5}{3}x + \frac{4}{3}) \times h(x)$
As $a_{n-2}x^2 + a_{n-1}x + a_n = k(x-1) \times (x - \frac{2}{3})$
$f(x) = 9x^2 - 15x + 6$ (also it has only 1 minima)
$\log(f(3) - f'(3)) = \log(10) = 1$

2. (8.00)
determinant of $M^{-1} = -adj(M)$

$$\begin{aligned} \frac{1}{|M|} &= -|M \times M| \\ |M| \times |M| \times |M| &= -1 \\ |M| &= -1 \\ |P \times P^T| \times |P^{-1}| &= \frac{|P| \times |P|}{|P|} \\ &= |P| \\ P &= -2M \\ |P| &= |-2M| \\ &= -8 \times |M| \\ &= -8 \times -1 \\ &= 8. \end{aligned}$$

3. (1.96)
Based on the inequality
AM≥ GM
So $\frac{(\frac{a}{7}+\frac{a}{7}+\frac{a}{7}+\frac{a}{7}+\frac{a}{7}+\frac{a}{7}+\frac{a}{7}+\frac{b}{2}+\frac{b}{2}+c)}{10} \geq a^{\frac{7}{10}} \times b^{\frac{2}{10}} \times c^{\frac{1}{10}} \times 3294172^{\frac{-1}{10}}$

So

$$\begin{aligned} \frac{14}{10} &= a^{\frac{7}{10}} \times b^{\frac{2}{10}} \times c^{\frac{1}{10}} \times 3294172^{\frac{-1}{10}} \\ \implies a^{\frac{7}{5}} \times b^{\frac{2}{5}} \times c^{\frac{1}{5}} \times 3294172^{\frac{-1}{5}} &= (1.4)^2 \\ &= 1.96 \end{aligned}$$

4. (16.63)
Here we split the area into three pieces :

$$A_1 = \int_1^2 (4x - \frac{4}{x})dx = 6 - 4\log 2$$
$$A_2 = \int_2^4 (\frac{16}{x} - \frac{4}{x})dx = 12\log 2$$
$$A_3 = \int_4^8 (\frac{16}{x} - \frac{x}{4})dx = 16\log 2 - 6$$

Thus the total area ,

$A = A_1 + A_2 + A_3 = 24\log 2$
$A = 16.632 = 16.63$ (approx)

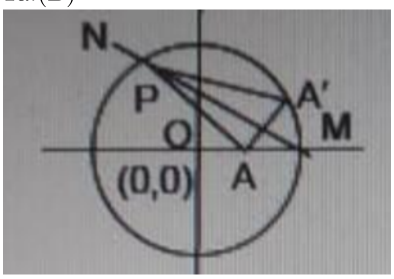
5. (3.00)
Split the circle at the fixed point and straighten it. According to the problem, the line is divided into 4 random parts, with the problem asking for the expected value of the sum of lengths of the first and last segments. This will be $\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$. Hence the required answer will be 3.

6. (1.00)
A is the region bounded by the square with vertices $1+i$, $1-i$, $-1-i$, $-1+i$. The curve is a line segment with Z_1 and Z_2 as endpoints. Here according to the condition it is the diagonal of the square. So, $Z_1 + Z_2 = 0$, $x = 0$

$$\begin{aligned}\sin i \log i &= \sin(i \log e^{\frac{i\pi}{2}}) \\ &= \sin(i \times \frac{i\pi}{2}) \\ &= \sin(-\frac{\pi}{2}) \\ &= -1 \\ c &= -1, d = 0\end{aligned}$$

Paragraph Comprehension

1a.(B)



Imagine a point A' on the circumference which coincides with A on folding. In the folded condition since A and A' are coincident, the distance of A and A' from any point on the crease line MN will be equal, i.e,

$|PA| = |PA'|$

where P(x,y) is any point on the crease line MN(we consider only the points within the circle). Also A = (a, 0) and A' = $(Rcos\alpha, Rsin\alpha)$(Parametric form of a point on the circle)

So we have:

$$\sqrt{(x-a)^2 + (y-0)^2} = \sqrt{(x - R\cos\alpha)^2 + (y - R\sin\alpha)^2}$$

which on solving yields:

$$x\cos\alpha + y\sin\alpha = \frac{R^2 - a^2 + 2ax}{2R}$$

Dividing by $\sqrt{x^2+y^2}$ on both sides:

$$\frac{x\cos\alpha + y\sin\alpha}{\sqrt{x^2+y^2}} = \frac{R^2 - a^2 + 2ax}{2R\sqrt{x^2+y^2}}$$

Let us define θ such that:

$$\sin\theta = \frac{x}{\sqrt{x^2+y^2}} \text{ and } \cos\theta = \frac{y}{\sqrt{x^2+y^2}}$$

Then we have $\frac{x\cos\alpha + y\sin\alpha}{\sqrt{x^2+y^2}} = sin(\theta + \alpha)$

So:

$$\sin(\theta+\alpha) = \frac{R^2 - a^2 + 2ax}{2R\sqrt{x^2+y^2}}$$

But we know for all β, $\sin\beta \leq 1$.
Therefore,

$$\sin(\theta+\alpha) \leq 1$$

$$\frac{R^2 - a^2 + 2ax}{2R\sqrt{x^2+y^2}} \leq 1$$

which on solving gives:

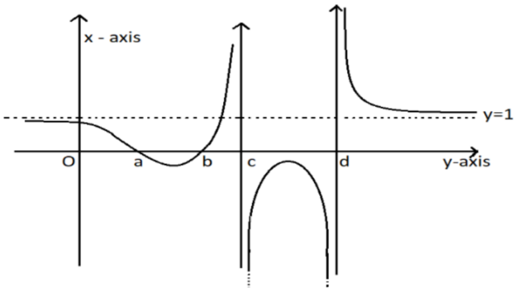
$$\frac{(x - \frac{a}{2})^2}{\frac{R^2}{4}} + \frac{y^2}{\frac{R^2 - a^2}{4}} \geq 1$$

Clearly the equation represents the region outside the ellipse with centre $(\frac{a}{2}, 0)$ and eccentricity given by:

$$e = \sqrt{1 - \frac{\frac{R^2-a^2}{4}}{\frac{R^2}{4}}} = \frac{a}{R}$$

1b. (A)

2.



2a. (B)
2b. (B)